Evaluation Formula for Fatigue Strength of Cruciform Welded Joints Failing from Weld Roots under Bi-Axial Loading

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Abstract

Fatigue strength and crack propagation behavior of cruciform welded joints in which fatigue cracks originate from weld roots have already been examined by a lot of studies under single axial loading condition. In the present study, fatigue strength evaluation formula for cruciform welded joints failing from weld roots under bi-axial loading condition has been proposed on the basis of the results of fatigue tests on 2 types of specimens with different weld penetration and fatigue crack propagation analyses on models with various plate thickness, weld size and weld penetration depth under several bi-axial loading conditions.

Keywords: Cruciform welded joint, root failure, fatigue strength evaluation formula, bi-axial loading, crack propagation analysis

1. Introduction

Fatigue failure in a steel structure is considered to occur at the portion with geometrical discontinuity and stress concentration such as welded joints in a steel bridge. There are many kinds of welded joints utilized in a steel bridge, and a typical one is a cruciform welded joint which connects steel plates or steel members. Fatigue crack origin in the joint consists of weld toe and weld root. Since it is difficult to detect the crack generated from the roots until it appears in the surface, the prevention of the root crack is important especially in view of maintenance. A lot of studies have been done about fatigue strength, fatigue crack initiation and/or propagation behavior of the cruciform welded joint failing from weld root (Maddox, 1991, Radaj, 2006).

Fatigue life is divided into crack initiation life (until a crack originates) and crack propagation life (after a crack originates). Most of fatigue life in case that cruciform welded joints fail from weld root is occupied by the crack propagation life. Therefore, fatigue crack propagation analysis based on fracture mechanics is admitted as a powerful tool for evaluating fatigue strength of cruciform welded joints in which fatigue cracks originate from weld roots. Frank and Fisher (Frank, 1979) indicated the empirical equation for stress intensity factor of fatigue crack generated from the weld root of a cruciform welded joint and conducted fatigue crack propagation analyses using the equation, then indicated that main factors affecting the fatigue strength were plate thickness, weld size and weld penetration depth. The author et al. examined influences of weld shape and above mentioned factors through fatigue crack propagation analyses and fatigue tests, and proposed the empirical equation for calculating weld throat thickness to arrange the fatigue strength using nominal stress range on weld throat sectional area (Mori, 2001). Furthermore, influence of root blowhole on fatigue strength (Mori, 1994) and critical weld leg length (Mori, 2000) were also made clear.

Above mentioned studies paid attention to single axial loading. However, there are not a few cases which are subjected to multi-axial loading due to a complicated structural detail and plate assembly in an actual structure. For example, corners of bridge steel bents are subjected to sectional force from beams and columns, and some fatigue cracks have appeared there in some bridges under severe truck loading conditions such as the Tokyo Metropolitan Expressway. Fatigue cracks with 30 mm long or more detected have been repaired by removing a core from the cracked portion and attaching a reinforced plate at the corner (Tokyo Metropolitan Expressway Technical Center, 2004). However, evaluation method for fatigue durability of the repaired corner has not been determined yet.

This study is aiming at constructing the numerical expression for evaluating fatigue strength of cruciform welded joints failing from weld root under bi-axial loading. For this purpose, fatigue tests were performed under bi-
axial loading using 2 types of specimens with different weld penetration. Furthermore, fatigue crack propagation analyses were also carried out on models with various plate thickness, weld size, and weld penetration depth under various bi-axial loading condition.

2. Specimens

The material used was structural steel SM490Y specified in the Japan Industrial Standard. The thickness of the steel was 36 mm. Mechanical properties and chemical compositions of the steel are indicated in Table 1. Three plates were cut out from the steel as the length was about 900 mm. These plates were assembled in a cruciform as shown in Fig. 1, and they were welded by CO₂ arc procedure under flat position. The welding conditions are shown in Table 2.

Specimens were divided into two types. One was the specimen with fillet welds (F-type specimen). Another was specimens with partially penetrated welds (P-type specimen). As for the P-type specimen, groove of 8 mm deep and 60 degree flank angle were machined on both sides of the plate.

After the welding was completed, specimens were sliced with 25 mm thick from the welded cruciform joint as shown in Fig. 1. Then these test pieces were subjected to post weld heat treatment in order to reduce welding residual stresses in condition that elevating speed of temperature is 70°C/hour, keeping at 620°C for 1.5 hours, and lowering speed of temperature was 70°C/hour. After the heat treatment, thickness of test pieces was reduced to 9 mm by milling machine. Examples of macroscopic photographs of specimen surfaces after etching are shown in Photo 1. Weld penetration depth of F-type specimen was ranged from 1.1 to 1.8 mm, and that of P-type specimens was from 9.0 to 9.9 mm (groove depth was included). Weld shape was close to an isosceles triangle and weld leg length was from 18.8 to 23.0 mm in each type of specimens. Configuration of specimen is shown in Fig. 3.

To measure residual stress, strain gauges of 1 mm gauge length were attached to specimen surface and cutting around the gauge as shown in Fig. 4. The results are shown in Fig. 5. The horizontal axis on the figure corresponds to the distance from the center of specimen.

<table>
<thead>
<tr>
<th>C</th>
<th>Si</th>
<th>Mn</th>
<th>P</th>
<th>S (%)</th>
<th>Y.P. (N/mm²)</th>
<th>T.S. (N/mm²)</th>
<th>Elongation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.16</td>
<td>0.48</td>
<td>1.55</td>
<td>0.14</td>
<td>0.003</td>
<td>395</td>
<td>555</td>
<td>32</td>
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</tbody>
</table>

**Figure 1.** Cruciform welded joint.

<table>
<thead>
<tr>
<th>No. of pass</th>
<th>Current (A)</th>
<th>Voltage (V)</th>
<th>Speed (cm/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>250</td>
<td>32</td>
<td>44</td>
</tr>
<tr>
<td>2</td>
<td>240</td>
<td>33</td>
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<td>3</td>
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<td>33</td>
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</tr>
<tr>
<td>6</td>
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</tr>
<tr>
<td>7</td>
<td>240</td>
<td>33</td>
<td>47</td>
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</tbody>
</table>

**Table 2.** Welding conditions

<table>
<thead>
<tr>
<th>No. of pass</th>
<th>Current (A)</th>
<th>Voltage (V)</th>
<th>Speed (cm/min)</th>
</tr>
</thead>
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<tr>
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<td>32</td>
<td>39</td>
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<tr>
<td>2</td>
<td>240</td>
<td>33</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>250</td>
<td>33</td>
<td>48</td>
</tr>
<tr>
<td>4</td>
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</tr>
<tr>
<td>7</td>
<td>250</td>
<td>33</td>
<td>57</td>
</tr>
</tbody>
</table>
Compressive residual stress of about 30 and 10 N/mm² is observed on un-welded portion in F-type and P-type specimen, respectively.

3. Fatigue Tests

3.1 Testing Method

Fatigue tests were performed under ten conditions in which cyclic loads ranging from 15.0 to 35.0 kN (load range 20.0 kN, stress range 61.7 N/mm²) or from 15.0 to 44.4 kN (29.4 kN and 90.0 N/mm²) were applied to the main plate, and cyclic load with -1, -0.5, 0, 0.5 or 1 times as much as load to main plate was applied to cross plate with the same phase using bi-axial fatigue testing machine (Mori, 2005). These values (-1, -0.5, 0, 0.5, 1) are called bi-axial stress ratio (\(\sigma_a/\sigma_m\), \(\sigma_a\); nominal stress on the cross plate, \(\sigma_m\); nominal stress on the main plate). The cyclic loading speed was set at 1.5 Hz. However, cyclic loading speed was 10 Hz when bi-axial stress ratio was equal to 0 (single axial loading condition).

Although stress relief heat treatment was performed as shown in the chapter 2, compressive residual stress of 10-30 N/mm² was measured. For the purpose of investigating the influence of these compressive residual stresses on the opening-and-closing behavior of weld root, the relationship between load and strain which is obtained from strain gauge striding the un-welded line as shown in Fig. 6. The measurement result of P-type specimen is shown in Fig. 7. From the load-strain relationship, opening-and-closing point of weld root is judged to be about 7.5 kN. As for the F-type specimen, it was about 5 kN. Minimum load in
fatigue tests was set at 15 kN on the basis of these results. The crack initiation and propagation during fatigue tests was observed by using digital microscope as shown in Fig. 8.

3.2. Test results

Some examples of the measurement results of a crack propagation route are shown in Fig. 9. In any case, the fatigue crack propagates in the direction which inclined to the main plate side to the direction of un-welded line. Relationship between fatigue crack initiation angle and

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Figure 5. Distribution of residual stresses.

Figure 6. Gauge location for measuring opening and closing behavior.

Figure 7. Opening and closing behavior of weld root.

Figure 8. Fatigue crack observation system.

Figure 9. Crack propagation routes.
bi-axial stress ratio is shown in Fig. 10. The definition of the initiation angle is also shown in this figure. The initiation angle is ranged from 1 to 25 degrees to the unwelded line direction. The angle becomes large with increase in bi-axial stress ratio. In addition, the angle obtained from F-type specimen is larger than one from P-type specimen.

The number of stress cycles (fatigue life) until fatigue failure occurs for each specimen is plotted in Fig. 11. Here the horizontal axis represents the bi-axial stress ratio. In each type of specimen and each stress range, the fatigue life becomes long with increase in the bi-axial stress ratio. In Fig. 12 shows the relationship between the normalized fatigue life and bi-axial stress ratio. The definition of normalized fatigue life is the ratio of fatigue life in any bi-axial stress ratio to one in which the bi-axial stress ratio is equal to 0. In both type of specimen, the relationship is roughly linear. The slope of the relationship is steep in P-type specimens in comparison with F-type specimens. That is, when the weld penetration is deep, the change of the fatigue life due to bi-axial stress ratio is large.

4. Stress and Fatigue Crack Propagation Analysis

4.1. Stress analysis

For the purpose of investigating the change of the stress by bi-axial stress ratio, elastic FEM analysis under two-dimensional plane strain conditions was conducted for the model whose weld size was 20 mm and weld penetration depth was 0. The analysis was conducted to 1/4 model in consideration of symmetrical configuration of the model. In this analytical model, unevenness of a welding bead surface is not taken into account, but is modeled as flat. The element used is 8 node one. The element size near the weld root was 0.5×0.5 mm.

Figure 13 shows contour diagrams of the stress in the direction of main plate when tensile stress (100 N/mm²) was applied to main plate (a), tensile stress (100 N/mm²) was applied to the cross plate (b), and compressive stress (-100 N/mm²) was applied to the cross plate (c). Fig. 13(a) reveals that high tensile stress has appeared near the
weld root tip. Compressive stress is observed near the root tip in Fig. 13(b), and tensile stress is obtained in Fig. 13(c). That is, tensile stress near the root tip due to tensile load of main plate increases when compressive load is applied to the cross plate, and decreases when tensile load is applied to the cross plate. This fact is considered to be the reason why fatigue life is influenced by bi-axial stress ratio.

4.2. Method of fatigue crack propagation analysis

General-purpose code for crack propagation analysis (FRANC 2D) (Swenson, 1997) which can perform re-division of the element according to crack propagation automatically was used for fatigue crack propagation analysis. The direction of fatigue crack propagation was determined using the maximum main stress theory in which the crack propagates in right-angled direction to the direction of the maximum main stress. J-integral method was used for calculation of a stress intensity factor K. The un-welded line is regarded as the initial crack. The critical crack length was set at $0.8 \times (\text{weld size}) + (\text{plate thickness})/2$. Design curve specified in the Fatigue design Recommendations issued by International Institute of Welding (IIW Recommendations) (IIW, 1996) is adopted as a relationship between fatigue crack propagation rate $\frac{da}{dN}$ and stress intensity factor range $\Delta K$.

$$\frac{da}{dN} = 3.0 \times 10^{-13} (\Delta K)^{3/2}$$

$da/dN$: mm/cycle, $\Delta K$: N/mm$^3/2$

Propagation analysis was conducted in the following procedures, and fatigue life was obtained.

(a) By using FEM, stress, strain, displacement and force at the node are obtained when arbitrary loads are applied to main plate and cross plate.

(b) After a fatigue crack has developed in the direction according to the maximum main stress theory, the same analysis as (a) is conducted. The extension of crack a step is set at 0.5 mm.

(c) J integral value is obtained from the result of (a) and (b), and transformed it into stress intensity factor range $\Delta K$.

(d) Substituting $\Delta K$ for the propagation law, the number of stress cycles required for a crack propagating from the condition (a) to the condition (b).

(e) Repeating (a) to (d) procedure until crack length reaches the critical size, fatigue life is obtained.

4.3. Comparison with experimental results

Analytical model simulating each specimen was constructed in consideration of weld penetration depth and weld leg length on the main plate side and cross plate side, and fatigue crack propagation analysis was conducted using FRANC 2D. However, unevenness of a weld bead face is not taken into consideration, but is regarded as flat. An example of the element division diagram near the crack tip is shown in Fig. 14. Relationship between crack initiation angles obtained from the analyses and experiments is shown in Fig. 15. Analytical results roughly agree with experimental ones.
Figure 14. Finite elements diagram near the crack tip.

Figure 15. Comparison of analytical results with experimental results (crack initiation angle).

Figure 16. Comparison of analytical result with experimental result (crack propagation route).

Figure 17. Comparison of analytical results with experimental results (fatigue life).

Figure 18. Illustration of analytical model.

5. Construction of Fatigue Strength Evaluation Formula

5.1. Analytical model

To examine influences of plate thickness T, weld size H, weld penetration depth Pw and bi-axial stress ratio \( \sigma_a/\sigma_m \) on fatigue strength of a cruciform welded joints failing from weld root, fatigue crack propagation analyses are conducted using the above-mentioned procedure and conditions. Illustration of analytical model is shown in Fig. 18. In these analyses, plate thickness T is set at 20, 30 or 40 mm, ratio of weld size to plate thickness H/T is set at 0.25, 0.50, 0.75 or 1.0, ratio of weld penetration depth to plate thickness Pw/T is set at 0, 0.2, or 0.4, and bi-axial stress ratio \( \sigma_a/\sigma_m \) is set at -1.0, -0.5, 0, 0.5 or 1.0. Fatigue life is obtained from the propagation analysis, and then it is transformed into fatigue strength at 2 million stress cycles using the relationship of fatigue life being inverse proportion to the 3rd power of a stress range. In these analyses, nominal stress range on the main plate is 100 N/mm². In addition, fatigue strength was arranged using the stress range on throat section according to the Fatigue Design Recommendations specified by the Japanese Society of Steel Construction (JSSC Recommendations) (JSSC, 1995). The throat thickness Th0 for obtaining a throat cross-section area is calculated.
from equation (2).

\[
\text{Th} = \frac{(H + Pw)}{2}
\]

(2)

5.2. Influence of plate thickness, weld size and weld penetration depth.

The author has already discussed fatigue strength of cruciform welded joints when fatigue crack originated and propagated from weld root using fatigue crack propagation analyses under the condition that fatigue crack propagation law was followed by the IIW Recommendations, fatigue crack extended in the direction of un-welded line through welded portion and stress intensity factor was obtained by energy method (Kainuma, 2006). Then, the following results have been obtained.

(1) The fatigue strength lowers in inverse proportion to the 1/6th power of the main plate thickness.

(2) The fatigue strength is almost constant regardless of the weld size in a region of the ratio of the weld size to plate thickness larger than 0.5. In a region of the ratio less than 0.5, the fatigue strength increases with decrease in the weld size.

(3) As the weld penetration deepens, the fatigue strength increases. This effect can be evaluated by applying the idea that plate thickness is reduced by the weld penetration depth (converted plate thickness \(T^* = T - 2Pw\)).

An example of the relationship between fatigue strength at 2 million stress cycles and plate thickness \(T\) which were obtained from the analyses here is shown in Fig. 19. In this figure, the weld penetration depth is set to 0 and the mark is distinguished by the ratio of weld size to plate thickness \((H/T = 0.25, 0.50, 0.75, 1.0)\). Fatigue strength is decreasing almost linearly in the log-log scale as plate thickness increases in every \(H/T\). The slope of these relationships is about \(-1/6\). In other models with different weld penetration, the same results have been obtained. Thus, this study considering crack propagation direction has arrived to the same conclusion as the former study (Kainuma, 2006).

Figure 20 shows the relationship between fatigue strength at 2 million stress cycles and weld size when plate thickness is 20 and 30 or 40 mm. Regardless of plate thickness, fatigue strength of \(H/T\) equal to 0.25 is higher a little bit, but fatigue strength where \(H/T\) is larger than 0.5 is almost constant. This conclusion is also the same as the former study (Kainuma, 2006).
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Figure 21 shows the relationship between fatigue strength at 2 million stress cycles and conversed plate thickness $T^*$. Fatigue strength decreases as conversed plate thickness increases. In a region where $T^*$ is less than 10 mm, the slope of the relationship is about $-1/4$, but the slope becomes gradual in a region where $T^*$ is larger than 10 mm. These results are different from ones obtained from the former study (Kainuma, 2006). Hereby, plate thickness (modified conversion plate thickness $T_s^*$) was re-calculated so that fatigue strength might decrease linearly in inverse proportion to the 1/6th power like the influence of plate thickness shown in Fig. 16. $T_s^*$ can be obtained from the equation (3).

\[
\frac{\text{(Fatigue strength at 2 million stress cycles of objective model)}}{\text{(Fatigue strength at 2 million stress cycles of a standard joint)}} = \left(\frac{20}{T_s^*}\right)^{1/6}
\]

(3)

The relationship between fatigue strength at 2 million stress cycles and modified conversion plate thickness is shown in Fig. 22. The slope of the relationship is about $-1/6$.

5.3. Influence of bi-axial stress ratio

Figure 23 shows the relationship between normalized fatigue strength and bi-axial stress ratio $\sigma_a/\sigma_m$ ($\sigma_a$: axial stress on cross plate, $\sigma_m$: axial stress on main plate). Normalized fatigue strength is defined as the ratio of fatigue strength under arbitrary bi-axial stress ratio to the fatigue strength under the ratio equal to 0 (single axial loading condition). In Fig. 23, the plate thickness is set at 20 mm and weld penetration depth is equal to 0, and the mark is distinguished by $H/T$. All the relationship between normalized fatigue strength and bi-axial stress ratio shown here have linearity. Therefore, the relationship can be expressed as follows.

\[
\text{Normalized fatigue strength} = \alpha \cdot \frac{\sigma_a}{\sigma_m} + 1
\]

(5)

$\alpha$ is constant for expressing the relationship between fatigue strength and bi-axial stress ratio, and the influence of bi-axial stress on fatigue strength is significant as $\alpha$ is high. The solid lines in the figure the regression line for the relationships of model with each $H/T$ value obtained by using least squares method. The variation of fatigue strength due to bi-axial stress ratio is remarkable when $H/T$ is large. Using these relationships, $\alpha$ which gives the variation of fatigue strength due to bi-axial stress ratio is expressed by equation (6).

\[
\alpha = 0.114(H/T)^{0.875}
\]

(6)

Figure 24 also shows the relationship between normalized fatigue strength and bi-axial stress ratio, in which the mark is distinguished by the ratio of the weld penetration depth to plate thickness (Pw/T). Fatigue strength has linear relationship with bi-axial stress ratio in any Pw/T, and slope of the relationship become steep with increase in Pw/T. This fact agrees with experimental results shown in Fig. 12. If the variation of fatigue strength due to bi-
Based on the throat thickness calculation formula (9) proposed here, fatigue lives are estimated in the condition that the fatigue strength is assumed as 42.3 N/mm², and they are compared with the fatigue lives obtained from the fatigue crack propagation analyses. The results are shown in Fig. 25 in which stress range on the main plate is set at 100 N/mm². In any model, the estimated fatigue life agrees with analytical one. Therefore, by using the equation (9), fatigue strength can be estimated in case that cruciform welded joints fail from weld root under bi-axial loading condition.

6. Conclusions

Fatigue crack propagation analysis on the basis of fracture mechanics conception and fatigue tests on model specimens were performed, and the influence of bi-axial loading on the fatigue strength is discussed. The main results obtained in this study are as follows.

(1) When tensile load acts on a main plate, tensile load acting on a cross plate extends fatigue life and compressive load shorten fatigue life. This fact was confirmed from fatigue test, stress analysis and crack propagation analysis.

(2) The influence of bi-axial loads is remarkable when weld size is large and weld penetration is deep. (3) By using stress range on weld throat sectional area obtained from the equation shown below, the influence of plate thickness, weld size, weld penetration depth and bi-axial loading on fatigue strength can be assessed.

\[
\text{Th} = \text{Th}1(\alpha \cdot \beta \cdot \sigma a/\sigma m + 1)
\]

(9)

Th1: equation (8), \(\alpha\): equation (6), \(\beta\): equation (7)

By determining weld throat stress range on the basis of equation (9) and comparing the stress range with the fatigue design curve specified, fatigue strength of cruciform welded joint failing from the weld root can be assessed in consideration of plate thickness, weld size, weld penetration depth and bi-axial stress ratio.

The problem that still remains is how much fatigue strength at 2 million stress cycles using weld throat section stress range obtained from the equation (9) should be determined. The fatigue strength at 2 million stress cycles of a basic joint (\(T=20 \text{ mm}, Pw/T=0, H/T=0.5\), single axial stress) obtained from fatigue crack propagation analyses was 42.3 N/mm². Moreover, the fatigue strength is specified as 40 N/mm² in the JSSC Recommendations and 45 N/mm² in the IIW Recommendations.

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