Steel Structures 6 (2006) 121-127

STEEL STRUCTURES
www.kssc.or.kr

On the Advanced Analysis of Steel Frames Allowing for Flexural, Local and Lateral-torsional Buckling

S. L. Chan[1,*] and C. K. Iu[2]

[1]*Department of Civil and Structural Engineering, The Hong Kong Polytechnic University, China*
[2]*Shenzhen Graduate School, Harbin Institute of Technology, China*

Abstract

Detailed procedure for second-order analysis has been coded in the newest Eurocode 3 and the Hong Kong steel code (2005). The effective length method has been noted to be inapplicable to analysis of shallow domes of imperfect members exhibiting snap-through buckling, to portals with leaning columns and others. On the other hand, the advanced analysis is not limited to buckling design of these structures. This paper demonstrates its application to the design of a simple plane sway portal and a three dimensional non-sway steel building. The results by the advanced analysis and the first-order linear analysis are compared and the technique for practical second-order analysis steel structures is described. It is observed that the use of a straight element by itself cannot model the buckling resistance of columns governed by different buckling curves for hot-rolled and cold-formed sections of various shapes like I, H, hollow etc. Also, the curvature of the conventional cubic Hermite element is not varied by the external axial force and thus it cannot simulate the response of a buckling column. Thus its use for second-order analysis is basically unacceptable. A technique for additional checking of beams undergoing lateral-torsional buckling is also suggested, making the advanced analysis a complete design tool for conventional steel frames.

Keywords: Lateral-torsional buckling, second order and advanced analysis, steel structures, frame analysis

1. Introduction

Advanced analysis is defined as an analysis allowing for second-order buckling and material plastic effects and imperfections in a frame such that checking of equilibrium and section capacity strength is adequate and individual member design is not needed. For clarity, the relationships between these analysis methods are plotted in Fig. 1. The relationship between various types of analysis is shown in Fig. 1. In the full second-order P-Δ-δ elastic and advanced analysis, the member resistance is directly checked against the code requirements. Research along this direction has been extensive over the past two decades. The two books on the subjects by Chen, Goto and Liew (1996) and Chan and Chui (2000) provide a detailed summary of the state-of-the-art method in non-linear analysis of steel frames.

Much research has been conducted on second-order analysis of steel frames. Most of the work assume straight element which cannot capture the difference in buckling strength for various sectional types like circular hollow sections and I-sections which use buckling curves “a” and “b” in the Eurocode 3 (2005). Research on frame stability design is mostly based on the cubic element which assumes a constant moment along a member. Chan and Zhou (1998) proposed the use of curved element allowing for member imperfection in design of steel frames. One should not be mis-percept that a common frame analysis program can handle a design by Advanced Analysis in fulfilment with the code requirements. The limitation of common software can be seen from its ignorance of member imperfections by using straight elements, use of inappropriate nonlinear solution methods and elements leading to early divergence and improper treatment of frame and member imperfection. In the design context, one cannot rely on a straight element itself to reflect member resistance in accordance with curves a_0 to d in

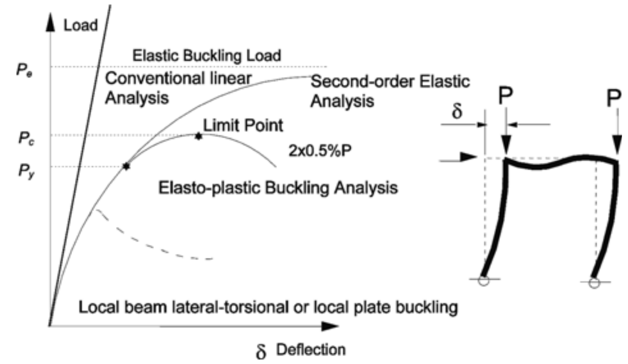

Figure. 1. Typical behavior of a steel portal.

*Corresponding author
Tel: +852-2766-6047, Fax: +852-2334-6389
E-mail: ceslchan@polyn.edu.hk

the Eurocode-3 (2005). Although use of several straight elements to model a member with calculated mid-node coordinates to simulate member off-set imperfection is theoretically possible to model an imperfect member, its use is too complicated and results in numerical divergence due to truncating error. It was noted that the approach is impractical and infeasible even for simple structures in practice.

This paper describes the concept and practical implementation of a second-order analysis for steel frames where column flexural, beam lateral-torsional and local plate buckling for slender cross section are all accounted for. It further indicates that the column flexural buckling is a system behaviour affected by stiffness of the complete frames and iterations for equilibrium are needed whereas other buckling modes can be considered at element level. Some researchers and engineers consider the inclusion of these principal buckling modes in a second-order analysis will lead to a change of design practice and it appears that the era for engineers in changing their practice is imminent.

1.1. Numerical methods for Linear and non-linear analysis

The first-order linear frames analysis is based on the matrix method of analysis and has been well documented decades ago and not repeated here. For non-linear analysis involved with the geometric and material nonlinearities, the incremental-iterative method is utilised. The tangent stiffness matrix is used for prediction of a load increment and the corresponding displacement increment is determined and accumulated to obtain the total displacement which is then used for finding the structural resistance. The incremental or residual force is computed from the difference between the structure resistance and the applied load and the complete process is repeated until the equilibrium error is sufficiently small after which another load increment is applied until the desired load level is attained or the structure collapses.

When plasticity is considered in the incremental-iterative matrix method of analysis, special care should be exercised to avoid divergence. The computer program, NIDA (2005), possesses an automatic function of dividing the load into smaller load increment controlled by the arc-distance when plastic design is activated. The yield surface for formulation of a plastic hinge is based on the yielding equation in code and when this condition is reached, the end springs of a member are then assigned a very small value for simulation of plastic hinges.

2. First Order Linear Analysis with Effective Length

In the effective length method, the critical problem for assessing the buckling strength will be the assumption of effective length. Unfortunately, the effective length of most columns cannot be estimated accurately in a simple way. The code suggests the use of a larger effective length, indicating the conservatism and uncertainty in effective length factor. Also, the actual behaviour of a frame allowing for various effects such as sway and change in member stiffness under high axial forces cannot be reflected in a linear analysis.

3. Second-order P-Δ-only Elastic Analysis

This is a common second-order analysis used in computer software for frame design and analysis and it refers to an analysis used to plot the bending moment and force diagrams based on the deformed or sway geometry. The method gives a result close to the hand amplification method and thus it is only limited to consideration of sway effects of a sway frame. It considers only the P-Δ effect but not the P-δ effect and checks the moment at member ends but not along members. Therefore one still needs to use design code to check the member strength, commonly based on an assumed effective length factor (L_E/L) equal to 1. Even allowing for all these, one cannot avoid the error associated with the change of member stiffness when it is under axial force. Thus, the method cannot provide a conservative design when a structure is susceptible to snap-through buckling. Further, its application to a non-sway frame is meaningless since the connection nodes do not sway, as in the second example in this paper.

4. Second-order P-Δ-δ Elastic Analysis

This analysis improves the above approach by further inclusion of P-δ effect so that the effective length for both sway and non-sway frames is determined in a computer analysis completely. The method stops at the first plastic hinge which is an assumption adopted in most design. It considers both the P-Δ and P-δ effects and named as the P-Δ-δ analysis. This term is to prevent confusion against those considering only one P-delta. Fig. 2 shows the

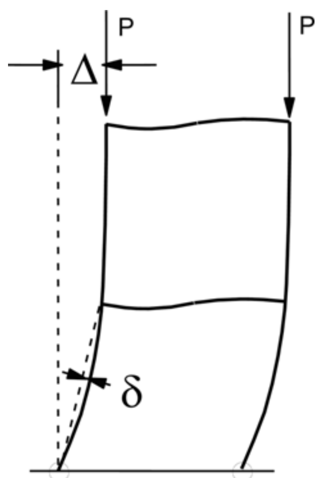

Figure 2. Consideration of P-Δ and P-δ effects with imperfections completely eliminates the effective length method.

concept in considering the buckling of a column and the effect from effective length can have been completely included when both the big Δ and small δ effects are considered.

5. Code Requirements in General

Reviews of some popular design codes in second-order analysis for design of steel structures show that codes require, either implicitly or explicitly, the inclusion of P-Δ and P-δ effects in the analysis or in the design, because they are inherent to all practical steel frames. For example, the use of moment amplification methods in BS5950 (2000) is for consideration of P-Δ effect and the use different buckling curves is for P-δ effect. In LFRD, the B_1 and B_2 factors are for P-Δ and P-δ effects respectively.

5.1. Concept of Second Order P-δ-Δ Elastic Analysis

Second-order analysis considers both the P-Δ and P-δ effects so that l_{cr} is not required to be calculated separately. The analysis and design procedure can be summarized as follows.

5.2. Section capacity check in a second-order P-δ-Δ analysis ignoring beam lateral-torsional buckling check

With other terms readily obtained from a linear analysis, the computer program should check the strength of every member by the following section capacity check.

$$\frac{P}{p_yA}+\frac{(M_y+P\Delta_y+P\delta_y)}{p_yZ_y}+\frac{(M_z+P\Delta_z+P\delta_z)}{p_yZ_z}=\varphi<1 \qquad (1)$$

where

P = axial force in member

p_y = design strength

Z_y, Z_z = effective modulus about principal axes

M_y, M_z = moment about principal axes

φ = material consumption factor. If $\varphi > 1$, member fails in design strength check and if $\varphi << 1$, waste of material since member strength can be reduced.

As its name implies, the analysis assumes the structure to behave elastically, although one can use the plastic modulus for determination of moment capacity. This approach is very often uneconomical since the failure of a secondary member in a redundant structure limits the load resistance of the complete frame. On the other hand, if an elastic analysis is used, one cannot scrutinise the effect of failure for the member over the whole frame and to determine whether or not the failed member is redundant and unimportant or key and critical member. To investigate the importance of the failure of a particular member, plastic analysis of tracing the structural response is needed and described as Advanced Analysis in the followings.

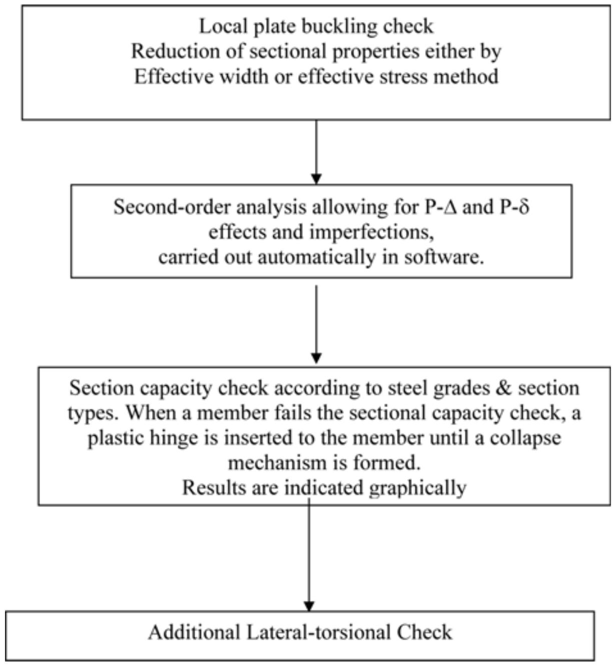

Figure 3. Procedure for second order P-δ-Δ analysis allowing for beam lateral-torsional buckling.

6. Advanced Analysis

To obtain a plastic collapse load or the large deflection plastic collapse load, a similar procedure as the above P-Δ-δ elastic analysis can be exercised, except that the analysis does not stop at the first plastic hinge. When a member is detected to have a plastic hinge with material factor φ in Eq. (1) greater than 1, the end spring of the member is then assigned a very small stiffness and the incremental-iterative process is continued until a plastic collapse mechanism is formed for the complete structure. The complete procedure allowing for lateral-torsonal and local buckling checks are indicated in Fig. 3.

6.1. Additional check for section capacity in second-order P-Δ-δ analysis allowing for beam lateral-torsional buckling check

So far, most advanced and second-order analysis is limited to design of structures without lateral-torsional buckling. Some researchers consider the second-order analysis has minimal impact to the engineering community until beam buckling is also considered in section capacity check. This paper proposes ideas of extending the advanced analysis to include the effects of lateral-torsional buckling under the design context.

For beam-buckling check of beams, an additional equation to Eq. (1) is developed by replacing the moment resistance M_c by buckling resistance moment M_b as,

$$\frac{P}{p_yA}+\frac{m_{LT}(M_y+P\Delta_y+P\delta_y)}{M_b}+\frac{M_z+P\Delta_z+P\delta_z}{p_yZ_z}=\varphi\leq 1 \qquad (2)$$

where m_{LT} is determined from the shape of bending moment diagram.

Note that the uniform moment fact, m_{LT}, is equal to 1 for worst scenario of beam under uniform moment along the length. The buckling design moment M_b can be obtained by conducting a beam buckling analysis based on the input boundary conditions and the plastic moment of the beam or, more directly and simply, obtained directly from the recommended value in the design code such as Appendix 8.1 in Hong Kong Steel Code (2005).

Equation (2) is generally taken as an additional check on top of the advanced analysis to prevent any early beam buckling before the frame collapse plastically. Numerically two options are suggested for dealing with the effect of lateral-torsional buckling. Specifically they are (1) to insert an additional hinge about the minor axis of the member undergoing lateral-torsional buckling and (2) to limit the frame design load to the load causing any member to undergo lateral-torsional buckling. The second approach seems to be a more engineering approach since beam buckling is generally considered unacceptable in engineering practice because of its brittle type of failure and difficulty in shedding loads on the buckled beam to other members. This phenomenon is rather different from a beam with plastic hinge of sufficient ductility to transfer excessive loads to other members.

For section local plate buckling check, either the effective width or the effective stress can be used. The calculation of effective width and sections can be carried out directly using code formulae and sectional dimensions. With the computed sectional geometry allowing for local buckling, Eq. (2) can be applied as a double checking procedure to Eq. (1).

7. Local Versus Global Effects

It can be seen iterations are needed to include the flexural column buckling effect whereas the direct formulae are used for beam lateral-torsional buckling effects. The rationale behind this is due to the system behaviour of column buckling. The local plate and lateral-torsional buckling of beams are localised effects and their checking in design codes is more on isolated members and therefore their design is simpler than the flexural column buckling. Column buckling is more a system interactive behaviour that its buckling strength is affected sensibly by member far away from it. As a result, frame classification is needed for column buckling check but the system interactive effect is ignored in design code for local plate buckling and beam lateral-tosional buckling checks. Thus, the present analysis considers column flexural buckling in a system behaviour and other knuckling modes as an individual member behaviour. However, second-order analysis allowing rigorously for beam lateral-torsional buckling can also be considered in a direct manner by methods proposed by Gu and Chan (2005) and others.

8. Member and Frame Imperfection

Most software for structural analysis and steel design do not consider member and frame imperfection in a rational manner. For example, codes require consideration of member imperfection and thus the use of curved element is a natural choice but most structural analysis software is still using the straight cubic element for second-order analysis.

In Table 6.1 of the HKSC (2005), a curved member with initial imperfection at mid-span denoted as δ_0 can be assigned by the users as,

δ_0/L=1/500 for curves “a”
=1/400 for curves “b”
=1/300 for curves “c”
=1/200 for curves “d”

9. Practical Examples

Two examples of advanced analysis are studied and compared with the first-order linear analysis. In the first problem, a plane portal frame is designed and analysed and in the second example, a three dimensional non-sway steel building is studied. All analysis and design are carried out by the software NIDA (2005).

Example 9.1. Plane portal frame

As shown in Fig. 4, the simple portal frame of height 10 m and width 30 m under a vertical point load of 1000 kN and a horizontal load of 60 kN at top is designed by hand method in Professor Trahair’s lecture note and by the present advanced analysis. All members are 356×368 ×153 H-section of cross area (A) of 195 cm^2 and second moment of area (I) of 48,500 cm^4 and modulus (S) of 2,680 cm^3. The objective of the design is to determine the ultimate design load factor.

When using the first-order analysis, the elastic critical load factor λ can be determined either by deflection method or by computer as 2.35. The bending moment at top of loaded column is 300 kN-m and thus the amplified moment is computed as $M\times\lambda/(\lambda-1) = 300\times2.35/1.35 = 522.2$ kN-m.

Using the buckling length of 1.0 L = 10 m, column

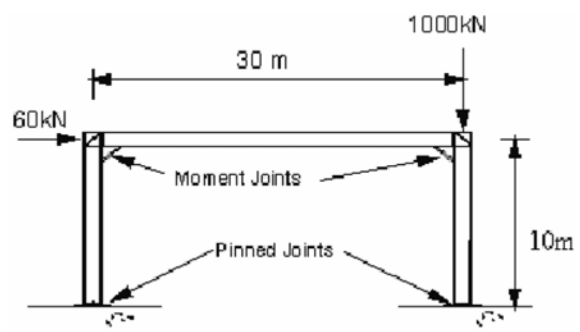

Figure 4. The simple portal frame.

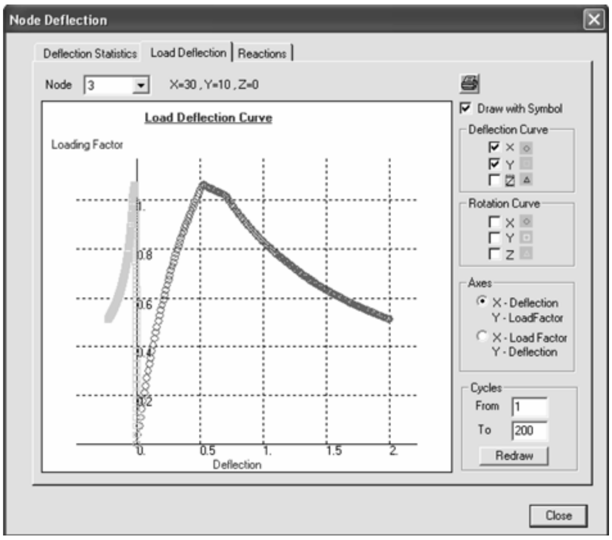

Figure 5. Load vs. deflection curve by advanced analysis for top node of the portal.

slenderness ratio = 10,000/158 = 63.3 and curve "b" in Table 8.8 of Hong Kong Steel Code (2005), the permissible buckling strength is 214.4 N/mm^2 and the permissible axial load P_c is = 19500×214.4 = 4180.8 kN

Combined Load Check:

$F/P_c + M/M_r = 1{,}000/4180.8 + 522.2/275/2680/10^{-3} = 0.948 < 1.0$

The design load factor is approximately 1/0.948 = 1.05.

When using the advanced analysis, the design load factor is obtained as 1.08 as indicated in Fig. 5. This indicates the reserve in strength after first plastic hinge is small because of low redundancy in the structure. The example further demonstrates the validity, accuracy and simplicity of the advanced analysis in dealing with the design of a sway portal. In the advanced analysis, the effective length of the column has not been assumed and the elastic buckling load factor is not required for computation. The efficiency for the complete design process is greatly improved.

Example 9.2. Design of a 3-dimensional non-sway frame

The steel building shown in Fig. 6 is adopted from MacGinley and Ang (1987) and the floor is under uniform pressure load as follows. Both the linear and the second-order analysis are used.

Figure 6. The 3-storey non-sway frame under floor loads.

Floor	Live load	Dead load
1st	3	7
2nd	3	7
Roof	1.5	5

Note: all units in kPa

A reduction of 10% live load is exercised for the column carrying floor loads for more than 1 storey. Thus a column on ground floor carrying 3 floors of loading is allowed to have 20% reduction in live load.

In the example, all beams are rigidly connected to columns and the frame is restrained from sway at all floor levels which can be achieved by tying to a core wall or other similar braced sub-frame. Columns at basement, first and second floors are respectively of sections 152×152×30 UC, 203×203×46 UC and 254×254×73 UC and steel grade S275.

In the linear analysis by NIDA, the design output is the same as the results by MacGinley and Ang (1987). In their analysis, the effective length is 0.85 of member length or floor height. The load resistance is calculated and compared against the applied loads to obtain the load factor shown in Table 1.

From the analysis output that the calculated design resistance of the frame is sensitive to the assumptions made for effective length and imperfections. For some members with very large imperfections or additional deflections due to loads along members, the effective length method cannot be used to reflect this characteristic.

In the P-Δ-δ analysis, no effective length for column

Table 1. Comparison of load factors by different methods and assumptions

Methods	Assumed Effective length factor (L_E/L)	Design Load Factor
MacGinley and Ang [5]	0.85	1.04
Member length as effective length	1.0	0.91
Chart method for finding effective length	0.68 for 2nd floor columns [4] 0.82 for other columns	1.11
Advanced analysis using imperfection factor e/L=1/400	Not needed	1.17

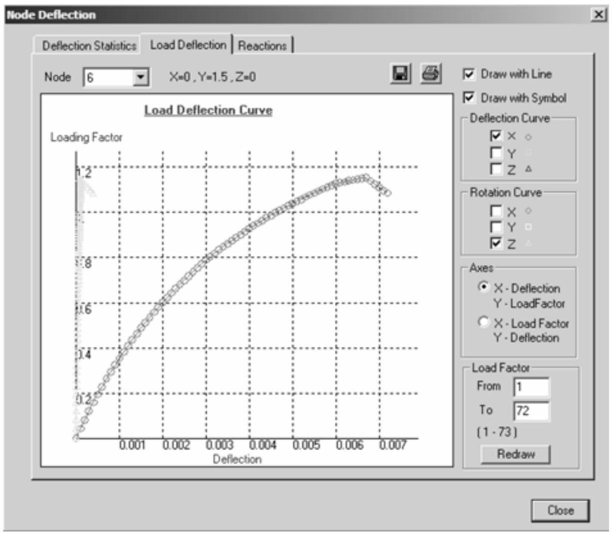

Figure 7. The load vs. deflection curve of the 3-storey frame.

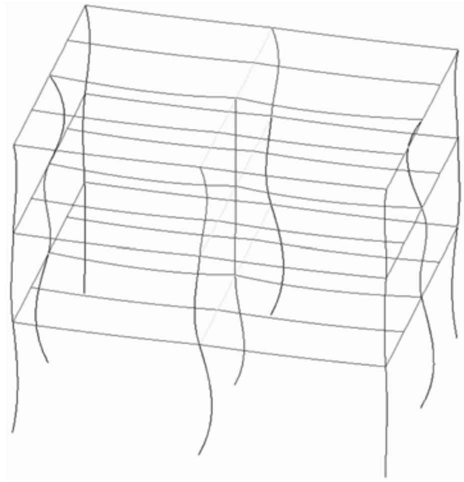

Figure 8. Deflected shape of the structure at ultimate design load factor of 1.18 (software: NIDA-version 7).

buckling is required and the frame is not needed to be classified as sway or non-sway frames. The structural behaviour is simulated and shown in Fig. 7. The effective length method gives different load resistance when varied effective length factor is used. Using the advanced analysis in the HKSC (2005), the result is close to the linear analysis by MacGinley and Ang (1987) with the effective length factor determined from the charts in Figure 6.4b of the HKSC(2000) with a difference of 5%. However, when the conventional effective length factor (L_e/L) of 1 is assumed, a 22% under-estimation of load capacity will be resulted. A distinctive advantage of the second-order or advanced analysis is the saving in designer's time and effort, in addition to its reliability in accurate computation of buckling resistance of every member rather than using an arguable effective length factor.

The analysis has also considered also the effect of lateral-torsional buckling of the vertical column. However, as the column buckles about its weak axis that the P-δ moment is larger about the weak axis, the reduction of resisting bending moment about the stronger axis due to lateral-torsional buckling has no effect on the load resistance of the column and the frame.

10. Conclusions

A planar portal and a two-bay three storey and 3-dimensional non-sway steel frame are analysed and designed by the first-order linear analysis method with the effective length approach and the second-order P-Δ-δ plastic analysis or advanced analysis allowing for member imperfections. It can be seen that the assumption of effective length factor (L_e/L) affects greatly the design resistance of the non-sway frame but its assumption can hardly be determined accurately in many practical cases. When using the proposed P-Δ-δ approach, the inconsistency does not exist and their interactive effect is considered by an iterative numerical process. The method has been used with success in a number of projects by technologically leading consultants and engineers in Hong Kong and Macau. The use of the approach is more reliable than the effective length method which cannot consider additional P-δ moment due to load along members and snap-through buckling. It is further estimated that the method is to be used in the newest collapse limit state design which places attention on collapse behaviour of a structure in stead of the considerations of the serviceability and ultimate limit states design.

Acknowledgement

The authors acknowledge the financial support of the Research Grant Council of the Hong Kong SAR Government on the project "Behaviour-Based Stability Analysis and Design of Slender Skeletons and Scaffolding Systems (PolyU 5135/03E)".

References

BS5950, British Standards Institution (2000) Structural Use of Steel in Building, Part 1, London, U.K.

Chan, S.L. and Zhou, Z.H. (1998) On the development of a robust element for second-order "Integrated Analysis and Design Approach", Journal of Constructional Steel Research, Vol. 47, No. 1/2, pp. 169-190.

Chan, S.L. (2005) "NAF-Nida: Nonlinear integrated design and analysis of frames, user's manual, 2nd edition, The Hong Kong Polytechnic University.

Chan, S.L. and Chui, P.P.T. (2000) Nonlinear static and cyclic analysis of steel frames with semi-rigid connection, Elsevier.

Chen W. F., Goto Y., and Liew J. Y. R. (1996) Stability Design of Semi-Rigid Frames, John Wiley & Sons, New York, pp. 468.

Code of Practice for the Structural Use of Steel, Hong Kong, 2005.

Eurocode 3 (2005) "Design of steel structures", Draft, DD ENV, 1993-1-1.

Gu, J.X. and Chan, S.L. (2005) "A refined finite element formulation for flexural and torsional buckling of beam-columns with finite rotations", Engineering Structures, 27, pp.749-759.

Load and Resistance Factor Design Specification for Structural Steel Buildings (2005) American Institute of Steel Construction, Chicago.

MacGinley, T.J. and Ang, T.C. (1987) Structural steelwork: design to limit state theory, Butterworth.